changed the parameter κ in (14). Similar data was obtained for other values of the determining parameters of the problem.

NOTATION

(r, ϕ, z), cylindrical coordinates; R, radius of the cylindrical ingot; v, withdrawal rate; T^*, phase transformation temperature; T_C, ambient temperature; T_0, initial temperature of the melt; λ, heat of phase transformation; k, thermal conductivity; c, heat capacity; α, coefficient of heat transfer with the environment; $Pe = vRc/k$, Peclet number; $St = \lambda/cT^*$, Stefan number; $Bi = \alpha R/k$, Biot number.

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THE STUDY OF NONLINEAR PROBLEMS OF HIGH-INTENSITY NONSTATIONARY HEAT TRANSFER

O. N. Shablovskii

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Analytic solutions are found of the nonlinear equations of heat transfer for a dominating effect of relaxation on the thermal flux evolution. The physical interpretation is given of the results obtained as applied to heat exchange problems in one-dimensional regions with moving boundaries.

1. Potential Systems of Equations of Heat Transfer. In the one-dimensional case the equations of heat transfer, in which the finite relaxation time of thermal flux is accounted for, are [1, 2]:

$$cT_t + q_x = 0, \tag{1}$$

$$\lambda T_x + \gamma q_t + q = 0. \tag{2}$$

We take into account that the following inequality is valid in a number of high-intensity nonstationary thermal processes [2-4]

$$|\gamma \partial q/\partial t| \gg q, \quad 0 \leqslant t \leqslant t_1 < \delta < \infty, \quad T \in [T_1, T_2], \tag{3}$$

making it possible to simplify the mathematical model (1), (2) and use in a considered δ-neighborhood of the initial moment of time the approximate equations

$$cT_t + q_x = 0, \quad \lambda T_x + \gamma q_t = 0. \tag{4}$$

The integral equation

$$q = \tau^{-1} \left[q^0(x) - \int_0^t (\tau \lambda T_x/\gamma)\, dt\right], \quad \tau = \exp(t/\gamma),$$

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following from (2) shows that the restriction (3) is valid, for example, for effects in which the wave property of transfer of thermal perturbations dominates [3-6], as well as for highly fluent processes, when $t_0 << \gamma$, where t_0 is the characteristic time.

For the thermophysical medium parameters we further consider various special variants of the dependences $\lambda = \lambda(T, t)$, $c = c(T, x)$, $\gamma = \gamma(T, t)$.

The potential [7] of the system of equations (4) can be introduced as follows:

$$v = \varphi_t, \; q = -\varphi_x, \; v(T) = \int_0^T [\lambda(T)/\gamma(T)]\, dT, \quad c = c(T, x),$$

$$\varphi_{tt} = w^2\varphi_{xx}, \quad w^2 = \lambda/c\gamma, \quad w = w(\varphi_t, x). \tag{5}$$

The equations of heat transfer (1), (2) correspond to the potential $\psi = \psi(x, t)$:

$$u = \psi_x, \; q = -\psi_t, \; u(T) = \int_0^T c(T)\, dT, \; \lambda = \lambda(T, t), \; \gamma = \gamma(T, t),$$

$$\gamma\psi_{tt} + \psi_t = (\lambda/c)\, \psi_{xx}. \tag{6}$$

Assuming that hypothesis (3) is satisfied in (6), we obtain

$$\psi_{tt} = w^2\psi_{xx}, \quad w = w(\psi_x, t). \tag{7}$$

2. Exact Solutions of the Equations of High-Intensity Heat Transfer (4). We apply the Legendre transformation $\Phi(v, x) = tv - \phi$ to Eq. (5):

$$\Phi_{vv}\Phi_{xx} - (\Phi_{vx})^2 = w^{-2}(v, x), \tag{8}$$

$$t(v, x) = \Phi_v, \quad q(v, x) = \Phi_x. \tag{9}$$

Subsequently, we reach the Monge–Ampere equation (8), investigated by the method of the intermediate integral [7, 8]. Using the mathematical results of this study, we find the exact solution of the heat transfer equations (4) in the parametric form (9).

If $w^2(v)$ is an arbitrary function, the solution is

$$t(v, x) = \pm xw^{-1} + \chi'(v), \; q(v) = \pm \int w^{-1} dv + \text{const}, \tag{10}$$

where $\chi(v)$ is also arbitrary. This implies that in the class of solutions (10) the specific thermal flux depends only on temperature, while the isotherms propagate at the rate of the thermal wave.

If

$$w^{-2} = f^2(\beta)(x + l_0)^{-4}, \quad \beta = (v + l_1)/(x + l_0), \quad l_0, \, l_1 - \text{const}, \tag{11}$$

then, according to [8], we find

$$t = \pm \frac{f(\beta)}{x + l_0} + \chi'(\beta), \quad q = \mp \frac{\beta f(\beta)}{x + l_0} + \chi(\beta) - \beta\chi'(\beta), \tag{12}$$

where $f(\beta)$, $\chi(\beta)$ are arbitrary functions. From (11) follows the important special case, when the thermophysical parameters λ, c, γ depend on temperature only:

$$f(\beta) = f_0\beta^{-2}, \quad c\gamma/\lambda = f_0^2(v + l_1)^{-4}, \quad v'(T) = \lambda/\gamma, \quad f_0 - \text{const}. \tag{13}$$

The restrictions (13) are satisfied, for example, by the functions

$$\lambda = \lambda_0 T^{n_1}, \quad c = c_0 T^{n_2}, \quad \gamma = \gamma_0 T^{n_3}, \quad T \in [T_1, T_2], \tag{14}$$

$$\lambda = \lambda_0 \exp n_1 T, \quad c = c_0 \exp n_2 T, \quad \gamma = \gamma_0 \exp n_3 T, \quad T \in [\hat{T}_1, \hat{T}_2], \tag{15}$$

where λ_0, c_0, γ_0 are arbitrary positive numbers, and n_1, n_2 are arbitrary. For the power dependence (14) we have $\ell_1 = 0$, $3n_3 = 3n_1 + n_2 + 4$, $c_0\lambda_0{}^3 = f_0^2\gamma_0^3(n_1 - n_3 + 1)^4$, while for the exponential (15)

$$l_1\gamma_0(n_1 - n_3) = \lambda_0, \quad 3n_3 = 3n_1 + n_2, \quad c_0\lambda_0^3 = f_0^2\gamma_0^3(n_1 - n_3)^4. \tag{16}$$

We now construct the solution of Eqs. (7). We apply to them the Legendre transformation $F(u, t) = xu - \psi$, and obtain the Monge–Ampere equation:

$$F_{uu}F_{tt} - (F_{ut})^2 = -w^2(u, t), \quad x(u, t) = F_u, \quad q(u, t) = F_t.$$

According to [8], for an arbitrary dependence $w^2(u)$ we have a solution similar to (10):

$$x(u, t) = \pm tw(u) + \sigma'(u), \quad q(u) = \pm \int w(u)\,du + \text{const}.$$

This simple class of thermal fields is not analyzed any further.

If

$$w^2(u, t) = f^2(\alpha)(t + k_0)^{-4}, \quad \alpha = (u + k_1)(t + k_0)^{-1}, \quad k_0, k_1 - \text{const}, \tag{17}$$

the solution is represented in the form

$$x = \pm \frac{f(\alpha)}{t + k_0} + \sigma'(\alpha), \quad q = \mp \frac{\alpha f(\alpha)}{t + k_0} + \sigma(\alpha) - \alpha\sigma'(\alpha), \tag{18}$$

where $f(\alpha)$, $\sigma(\alpha)$ are arbitrary functions. The relationship (17) makes it possible to isolate the special case, when λ, c, γ depend on temperature only:

$$f = f_0\alpha^{-2}, \quad \lambda/c\gamma = f_0^2(u + k_1)^{-4}, \quad u'(T) = c(T), \quad f_0 - \text{const}. \tag{19}$$

The restriction (19) is obeyed, for example, by the functions (14), for which $k_1 = 0$, $n_3 = n_1 + 3n_2 + 4$, $\gamma_0 f_0^2(n_2 + 1)^4 = \lambda_0 c_0^3$, as well as functions (15) if $k_1 = c_0/n_2$, $n_3 = n_1 + 3n_2$, $\gamma_0 f_0^2 n_2^4 = \lambda_0 c_0^3$.

Thus, for the approximate equations (4) we found the two exact analytic solutions (12), (13) and (18), (19), containing one arbitrary function of a single argument. The sign $\pm$ in these solutions refers to the two different families of characteristics of the system (4); from now on we choose everywhere the upper sign. The sign of the number f_0 is selected according to the specific conditions of the problem.

3. Nonlinear Heat Exchange at a Wall. We provide the physical interpretation of the solution (12), (13). Let the relaxing thermal field be determined by the dependences

$$\lambda T_0'(x)/\gamma = \beta_0, \quad q_0 = -\beta_0 t, \quad v(T_0) = v_0(x), \quad T_0 < \infty,$$
$$\beta_0 \equiv \text{const}, \quad x_w \leqslant x < x_1 < \infty, \quad t \in [0, t_1], \tag{20}$$

satisfying Eqs. (1), (2) within the approximation (3) adopted.

Consider a process flowing in the region between the immobile wall $x = x_w$ and the front of the thermal wave $x = x_0(t)$, $x_0(0) = x_w$, propagating along the background (20):

$$x = x_w = 0:\ q = q_w[v(T_w)]; \quad x = x_0(t):\ v = v_0, \ q = q_0.$$

These boundary conditions correspond to the fact that heat exchange at the wall occurs by a given nonlinear law, while the temperature and the specific thermal flux are continuous at the wave front. The solution (12), (13) satisfies the conditions mentioned if

$$\chi(\beta) = \beta\left[\frac{f_0}{2l_0}(\beta^{-2} - \beta_0^{-2}) - \int_{\beta_0}^{\beta} \beta^{-2}R(\beta)\,d\beta\right], \tag{21}$$

where $\ell_0 = -x_1 < 0$ is a numerical parameter. The function $R(\beta)$ is obtained from $q_w(v_w)$ by replacing the argument v_w by $\ell_0\beta - \ell_1$; $R(\beta) = q_w(\ell_0\beta - \ell_1)$ in which case it must be bounded $|R(\beta)| < \infty$, and such that the algebraic equation $R(\beta_0) = 0$ have a real root $\beta_0 \neq 0$. For example, if $q_W = q_1 + q_2 T_W^b$, then for the variant (14) this function exists for $b(n_1 - n_3 + 1) < 0$. Depending on the sign of the number β_0, the solution is constructed in the region $\beta \leq \beta_0 < 0$ or $\beta \geq \beta_0 > 0$. Taking into account that each line of the family β = const propagates at the rate of the thermal wave, we select $\beta = \beta_0$ and find the equation of motion:

$$x_0(t) + l_0 = f_0 \beta_0^{-2} [t - \chi'(\beta_0)]^{-1}, \quad \chi'(\beta_0) = -f_0/l_0\beta_0^2 < 0, \quad f_0 < 0,$$

which depends on the thermophysical properties of the medium and of the parameter β_0, characterizing the relaxing thermal background (20). For increasing β_0 the thermal wave velocity decreases monotonically. The front of the thermal wave is found initially at the wall, and for $t > 0$ it propagates to the right with negative acceleration. Heat transport is described by the solution (12), (13), (21), and a simple equation is obtained for the specific thermal flux:

$$q(v, x) = \frac{f_0 x}{l_0 (v + l_1)} + q_w \left(\frac{l_0 v - l_1 x}{l_0 + x} \right),$$

providing a smooth representation on the nature of its dependence on the boundary conditions.

4. Strong Breakdown of Thermal Field. Based on the solution (12), (13) we consider a thermal field, containing a strong breakdown $x = x_j(t)$. Studies of the nature of the breakdowns generated in nonlinear media with thermal relaxation and a bibliography of studies on this subject are contained in [2, 4, 6]. The conditions for strong breakdown of the thermal field were analyzed in [9, 10], and they are

$$q_j - q_j^* = N(u_j - u_j^*), \quad \Lambda_j - \Lambda_j^* = \gamma N (q_j - q_j^*), \quad N = x_j'(t), \quad \gamma \equiv \text{const}. \tag{22}$$

We choose the variant (15), (16), assuming $n_3 = 0$, $n_2 = -3n_1 > 0$. The qualitative behavior of these dependences ($\gamma \equiv$ const, $c'(T) > 0$, $\lambda'(T) < 0$) correspond, for example, at temperatures from 1.8 to 2.1°K to thermophysical properties of liquid helium [11], in which second sound shock waves can be generated [6, 12]. Several results of the study of experimental data [11] were given in [5].

For $t = 0$ let there by strong breakdown at the point $x = 0$. On its right the medium temperature is constant: $v(T^*) \equiv$ const, $q^* \equiv 0$, and on the left is located the thermal field $v^0(x)$, $q^0(x)$, $x \in [-H, 0)$, continuously passing into the "cold" background $T \equiv 0$, $q \equiv 0$, $x \leq -H < 0$. We assume that $T_j^0 < T^*$, so that at $t > 0$ a cooled shock wave propagates to the right, and a continuous heated wave – to the left [9]. We construct the solution for $x \in [x_0, x_j]$, $t > 0$.

Methodologically the given problem is similar to the gas dynamics problem of decay of an arbitrary rupture [7].

Satisfying the conditions at the wall (22), we find

$$\chi(\beta) = \beta \left(C_2 - \int_{\beta_j^0}^{\beta} \frac{2f_0 + C_1\beta}{\beta^2 (V + l_1)} d\beta \right), \quad \beta \in [\beta_0, 0), \tag{23}$$

$$\begin{gathered}
[V(\beta) + l_1]^3 = m_1 - m_2 (f_0 + C_1\beta)^2 < 0, \quad l_1 = \lambda_0/\gamma n_1 < 0, \; v_j = V(\beta_j), \\
N = (v_j + l_1)^2/\varepsilon_j, \quad q_j = \varepsilon_j/(v_j + l_1), \quad \varepsilon_j = f_0 + C_1\beta_j, \\
m_1 n_2 = m_2 c_0 l_1^3, \quad m_2 (c_0 + u^* n_2) = n_2, \quad f_0 > 0, \\
\beta_j'(t) = 3\beta_j^2 v_j^4 \varepsilon_j^{-1} [m_2 \varepsilon_j (2f_0 + \varepsilon_j) - 3m_1]^{-1} > 0, \\
\beta_j^0 \equiv \beta_j(0) = (v_j^0 + l_1) l_0^{-1} < 0, \quad \beta_0 < \beta_j^0 \leqslant \beta_j(t) < 0.
\end{gathered} \tag{24}$$

The integration in (23) is carried out on both sides up to the value β_j^0. From the breakdown stability conditions [7] $w_j < N < w^*$ we obtain the estimate

$$f_0 > -\beta_j^0 C_1 > 0, \; C_1 > 0, \; (m_1 - m_2 f_0^2)^{\frac{2}{3}} < f_0 w^*. \tag{25}$$

The initial jump $v_j^0 - v^*$ must be assigned with account of (25) and the inequality $v_j^0 + \ell_1 < 0$.

The equation of motion of the thermal wave front $\beta = \beta_0 \equiv \text{const}$ is expressed by the equation

$$x_0(t) + l_0 = f_0\beta_0^{-2}[t - \chi'(\beta_0)]^{-1}, \quad l_0 = H - (f_0/\beta_0^2\chi'(\beta_0)) > 0,$$
$$H = -x_0(0) > 0.$$

A consequence of the continuity of T, q at $x = x_0(t)$ is the equality $f_0 + C_1\beta_0 = 0$ making it possible to calculate β_0. From the condition $x_j(0) = 0$ we obtain

$$C_2[(v_j^0 + l_1)\beta_j^0]^2 = (f_0 + \varepsilon_j^0)(v_j^0 + l_1) - f_0 l_0 (\beta_j^0)^2 < 0,$$

in which case the inequality $\chi'(\beta_0) < 0$ is satisfied.

The constants H, C_1 characterize the initial thermal field left of the rupture by means of the dependences $t(v^0(x), x) = 0$, $q(v^0(x), x) = \chi(\beta^0)$, x [$-H$, 0).

The solution of the problem stated is given in parametric form by Eqs. (12), (13), (23), (24). The vector of specific thermal flux is directed toward heating, which is slowly displaced. With the flow of time the temperature at the breakdown front decreases monotonically, and the absolute value of thermal flux increases. In the case $E_j^0 \equiv m_2(\varepsilon_j^0)^2 + 3m_1 > 0$ the cooled shock wave moves with acceleration at $t > 0$. If the thermophysical parameters of the medium and the initial conditions of the problem are such that $E_j^0 < 0$, then there exists a finite value t_2, determined by the condition $E_j(t_2) = 0$, for which the rate $N(t_2)$ is minimal. For $0 \leq t < t_2 < \infty$ the motion of the shock wave is slowed down, while for $t > t_2$ it is accelerated.

5. Nonstationary Melting of a Solid. Thermal processes of nonstationary melting of material under the action of intense energy fluxes were investigated on the basis of the parabolic equation of heat conductivity ($\gamma \equiv 0$) in many studies [13]. Most results were obtained under the condition that the effect of the liquid phase on heat transport to the solid is insignificant. A more rigorous analysis requires simultaneous treatment of thermal effects in both phases [14]. A review of studies on the problem of accounting for the finite propagation velocity of heat during phase transformations is provided in [13]. It is noted that in the case of melting of metals the thermal relaxation is manifested for sufficiently high values of thermal flux density at the surface.

We apply the solution (18), (19) to the problem of melting of material under the action of a surface heat source for moments of time near the initial moment, neglecting the effect of layer decay. Consider the process between the melting boundary $x = x_m(t)$ and the front of the thermal wave, propagating according to the relaxing thermal field:

$$u(T_0) \equiv u_0(t) = \alpha_0 t + u^0, \ \alpha_0 > 0, \ u_0(t_1) = Au_m, \ 0 < A < 1, \ t \in [0, t_1],$$
$$q_0(x) = b_0 - \alpha_0 x, \ -\infty < -H_1 \leqslant x \leqslant -H_0 < 0, \ \alpha_0, \ b_0, \ u^0 - \text{const}.$$

These dependences satisfy Eqs. (1), (2) within the approximation adopted (3). The boundary conditions are

$$x = x_m(t): \ q = \tilde{q} - L(\gamma x_m'' + x_m'), \quad u = u_m \equiv \text{const}, \tag{26}$$
$$x = x_0(t): \ q = q_0, \quad u = u_0. \tag{27}$$

Here $\tilde{q} = \tilde{q}(t + k_0)$ is an arbitrary analytic function.

The family lines $\alpha = \text{const}$ propagate with the velocity w of the thermal wave; therefore, choosing $\alpha = \alpha_0 > 0$, $u^0 = \alpha_0$, $k_0 - k_1 > 0$, we find the equation of motion of its front $x_0(t) = [f_0/\alpha_0^2(t + k_0)] + \sigma'(\alpha_0)$, $k_0 > 0$. The continuity conditions (27) will be satisfied if $\sigma(\alpha_0) = b_0$. In the solution (18), (19) we choose $u(T_m) = u_m$, and determine the equation of motion of melting boundary

$$x_m(t) \equiv x(u_m, t) = \frac{f_0}{\alpha_m^2(t + k_0)} + \sigma'(\alpha_m), \quad \alpha_m = \frac{u_m + k_1}{t + k_0}.$$

Initially the energy is observed if we take into account the approximation (3), i.e., we omit the term $Lx'_m(t)$. For the function $\sigma(\alpha)$ we then obtain a third order linear inhomogeneous differential equation, whose general solution is represented by the equations:

$$\sigma(\alpha)=\alpha\int_{\alpha_m^0}^{\alpha}\Psi(\alpha)\,d\alpha+C_3,\quad \Psi=R_1\Psi_1+R_2\Psi_2,\ \alpha\in[\alpha_0,\ \alpha_m^0], \tag{28}$$

$$\Psi_1(\alpha)=\alpha^{-2}I_0(z),\ \Psi_2(\alpha)=\alpha^{-2}K_0(z),\quad z=2u_m(\alpha L\gamma)^{-\frac{1}{2}},$$

$$R_1=C_1-2S_1,\ R_2=C_2+2S_2,\ C_1,\ C_2,\ C_3-\text{const},$$

$$S_1(\alpha)=\int_{\alpha_m^0}^{\alpha}\alpha^3S(\alpha)\,K_0(z)\,d\alpha,\ S_2(\alpha)=\int_{\alpha_m^0}^{\alpha}\alpha^3S(\alpha)\,I_0(z)\,d\alpha,$$

$$S(\alpha)=u_m^2\,[\tilde{Q}(\alpha)-C_3+(f_0/u_m)]/L\gamma_m\alpha^5,\ \tilde{Q}(\alpha)=\tilde{q}\,[(u_m+k_1)/\alpha].$$

Here $\tilde{Q}(\alpha)$ is obtained from $\tilde{q}(t+k_0)$ by replacing the argument $t+k_0$ by $(u_m+k_1)/\alpha$; and $I_0(z)$ and $K_0(z)$ are the zeroth order modified Bessel functions of the first and second kind.

Let $f_0>0$, $x_m(0)=0$, $x_0(0)=-H_0<0$, the thermal wave and the melting boundary propagate to the left, toward negative x values. Taking into account that $u_0<u_m$, $0<\alpha_0<\alpha_m\leq\alpha_m^0$, from the conditions

$$x_0(t)<x_m(t)\leqslant 0,\quad x_0'(t)<0,\quad x_m'(t)<0,\quad t\in[0,\ t_1]$$

we obtain the estimates:

$$\sigma''(\alpha)>0,\ \sigma'''(\alpha)<0,\ 0<z(\alpha)\leqslant 4,\ \alpha\in[\alpha_0,\ \alpha_m^0],\ \alpha_0>u_m^2/4L\gamma,$$

$$C_3>\tilde{Q}_1+(f_0/u_m),\quad \tilde{Q}(\alpha)\leqslant\tilde{Q}_1<\infty,\ H_1>-x_0(t_1),$$

$$B_1=\frac{B(1-A)\,u_m}{\alpha_0k_0\,(Au_m+k_1)}\left[1-\frac{(Au_m+k_1)^2}{(u_m+k_1)^2}\right],$$

$$B_2=\frac{B\,(Au_m+k_1)^2}{\alpha_0^2\,(u_m+k_1)^3},\quad B=-\frac{2f_0}{z_m^0}\,(\alpha_m^0)^2,$$

$$\frac{C_1}{z_m^0\,K_0(z_m^0)}+\frac{f_0K_1(z_m^0)}{(u_m+k_1)\,K_0(z_m^0)}<\min\{B_1,\ B_2\},$$

$$C_2K_0(z_m^0)=\frac{-f_0}{u_m+k_1}-C_1I_0(z_m^0)>0.$$

The last two inequalities are easily satisfied by the choice $C_1<0$. The thermal field between the region boundaries is described by the solution (18), (19), (28), and at the initial moment of time they depend on the constants $u^0>0$, $k_0>0$, $C_1<0$, $C_3>0$. The first equation in (18) is convenient for finding isotherms in the melting zone of the solid. The specific thermal flux at the melting boundary increases monotonically for $t\in[0,\ t_1]$; the front of the thermal wave moves very slowly.

The examples considered of nonlinear problems of heat transfer show the effectivenes of using the analytic solutions (12), (13) and (18), (19) in high-intensity nonstationary processes, in which the effect of thermal flux relaxation is predominant.

NOTATION

x, Cartesian coordinate; t, time; T, temperature; q, specific thermal flux; λ, thermal conductivity coefficient of the medium; c, specific bulk heat capacity; γ, relaxation time of thermal flux; w, propagation velocity of small thermal disturbances; L, phase transition latent heat per unit volume of the material; $\tilde{q}(t)$, assigned density of thermal flux at the surface; and $\Lambda'(T)=\lambda(T)$; $E_j=m_2\varepsilon_j^2+3m_1$. Subscripts: 0 (subscript), value of a function at the initial moment of time; 0 (superscript), a thermal field parameter ahead of the thermal wave front $x=x_0(t)$; *, thermal field parameters ahead of a strong discontinuity; j, m, w, functional values at the line of strong discontinuity $x=x_j(t)$ at the melting boundary $x=x_m(t)$, and at the wall $x = x_w$; a prime above a function is ordinary differentiation; and independent variables as subscripts denote partial derivatives.

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PRACTICALLY ACHIEVABLE ACCURACY AND RELIABILITY OF THE SOLUTION OF INVERSE HEAT-CONDUCTION PROBLEMS

P. I. Balk

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We develop a method of solution of inverse heat-conduction problems which makes it possible to obtain a guaranteed minimum of reliable information in conditions of indeterminacy.

In practical analyses of data from model and natural thermal experiments, wide attention has been given to methods based on the solution inverse heat-conduction problems (IHCP) with the use of regularization [1]. It is known, however, that the regularization theory gives only a potential possibility to solve incorrectly posed problems. The regularization method itself has an asymptotically optimal character when the quality of the approximate solution is estimated from its behavior in comparison with the exact solution when the error of observations tends to zero. If the number of measurements is small and the noise is appreciable, the convergence of approximate solutions is of secondary importance, and the principal problem is to extract the maximum amount of reliable information from the available data, and to isolate fragments of solution which, under the existing indeterminacy, are observed reliably.

This formulation of the problem must be viewed alongside the fact that, in realistic conditions, there are always sufficiently large regions of competing interpretations of the input data (which are, objectively, of equal value) and any "optimum" solution chosen according to some principle, is capable of adequately reflecting only individual fragments of the true solution. It is difficult to analyze reliability of the local properties of the approximate solutions in terms of the classical estimates of accuracy constructed in terms of the metrics L_p. These facts stimulate the development of applied methods (adaptive [3], descriptive [4], local [5] and stepwise [6] regularizations) which make it possible to narrow down maximally the mass of the permissible solutions of the inverse problem by virtue of a more complete allowance for the restrictions on the properties of solutions and noise, and of a more special

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